How can recurring patterns and forms found in nature be explained mathematically and why are humans visually drawn to recreating these patterns?

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Abstract

Repetitive patterns are commonly found in nature and in human creations. In this research I studied pattern, mainly focusing on the role of the Fibonacci sequence in plant structures and the aesthetic value of this sequence. Why humans are drawn to recreating natural patterns is also explored. This research looks at studies conducted from mathematical and physiological perspectives, exploring both recent and past experiments. It was found that Fibonacci patterns and other self-assembling patterns can be used in the field of stress engineering and as a tool in the medical field. It was concluded that humans are overall drawn to these patterns because they prove to be helpful in design. Research on theories behind why humans are visually drawn to these patterns was also done. The most significant results are that patterns affect brain functionality by being proportional to our field of vision and by slowing visual processing. This research is just the beginning of an exploration, more evidence is still needed to understand the full aesthetic value and implications of the Fibonacci sequence and other natural patterns in our world.

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Part I: Introduction

Walking through a garden, light flutters down from a tree above. Stems extend towards the sun. Branches expand as you gaze down towards the heart of the tree. The roots underground mirror the tree above, absorbing water instead of air. Water is pushed up the trunk, through the branches, into the stems, to nourish the leaves. Flowers hug the base of the tree sprouting leaves spiraling towards the head where seedlings start their spinning journey. The petals count 5, 8, and 13. If you were to walk through Claude Monet's "Le Jardin de l'Artiste à Giverny (1902)" you may see this sight, a masterpiece of nature within a work of art.

Humans strive to better understand themselves and the world around them. Beauty can be found in the natural environment and in human creations. Certain patterns embed themselves throughout the world and within human bodies. Is there a collective law which connects the eye of a hurricane, the arm of a galaxy, the beat of a heart, and the intricate petals of a flower? Humans are pattern seekers. What humans observe in nature is mimicked in their designs in different fields such as art, architecture, music, and mathematics. Humans seek beauty and are drawn to the beautiful recurring patterns seen. Aesthetic appeal found in nature helps us strive to advance and better understand ourselves and the world around us.

Some patterns found in nature can be explained through the Fibonacci sequence, the constructal law and the evolution of flow. Humans are drawn to these patterns and forms because they can cause an aesthetic reaction by affecting the brain's ability to process information and can serve as a tool in the world.

Part II: Historical Context/Background

The Fibonacci sequence was first introduced to the Western world by Leonardo of Pisa, an Italian mathematician, also known as Fibonacci. Although Leonardo made many contributions to the field of mathematics, he is known for his use of the Fibonacci sequence in his work. He first used this series in his book *Liber Abaci*(1202) where the sequence was used to illustrate the population growth of rabbits (Grigas 6). The series starts as 0, 1, 1, 2, 3, 5, 8, 13…

The following number in the sequence is found by finding the sum of the previous two numbers. From this series we can obtain the golden ratio, or the irrational number of phi (ϕ .) By multiplying the numbers in the sequence together, the area of a golden rectangle is found and by dividing the dimensions of these rectangles the solution approaches the golden ratio as shown in Figure 1 (Grigas 6).

Dimensions of a few golden rectangles:	When these dimensions are divided:
8 × 13	13 ÷ 8 = 1.625
13 × 21	21 ÷ 13 = 1.615...
21 × 34	34 ÷ 21 = 1.619...
34 × 55	55 ÷ 34 = 1.6176...
55 × 89	89 ÷ 55 = 1.61818...

The golden ratio (ϕ)= 1.6180339887…..

Figure 1

A more exact representation of this number can be shown through the equation:

$$\frac{(1+\sqrt{5})}{2} = 1.6180339887\ldots$$

It is important to note that when referring to either the golden ratio or the Fibonacci sequence one must acknowledge seeing their connection. Addressing a “golden” term such as golden rectangle, golden angle, golden section… entails that these forms can be derived from the Fibonacci sequence.

For example, the golden angle (ψ) can be found through an equation where the reciprocal of phi is multiplied by 360º and the result is then subtracted from 360º (Grigas 20).

$$\psi = (360^{\circ} \times (1/\phi)) - 360^{\circ} = 137.5077640501 \approx 137.5^{\circ}$$

Numbers in the Fibonacci sequence manifest themselves in many forms across the natural world, such as the design of plants, animals, and even galaxies. The same numbers can also be uncovered in various forms of human creations such as art, architecture, music, and mathematics. The patterns and ratios that can be derived from this series has been theorized to hold a profound beauty to the human eye. The desire to better understand humanity and the world has driven the exploration of this topic. Scientists, artists and mathematicians have used and explored the concept of the golden ratio and Fibonacci sequence for centuries.

Part III: Research and Analysis

The Constructal Law

A notable professor in the engineering and applied physics field, Adrian Bejan, is known for his contributions to the concept of the constructal law and has defined its role in nature. In his book, *Design in Nature,* he states that the constructal law accounts for natural form, design, and evolution. Bejan argues that structures in nature (inanimate and animate) have a tendency to facilitate flow: using the least amount of energy needed to perform a certain task (3). For example, an organism will facilitate flow through the movement of nutrients through its system. Flow systems have evolved to become as efficient as possible since evolution favors energy efficient systems. An important property of a flow system is that it can grow and morph in order to obtain easier access to flow. This is defined as being different than a static system, which is fixed and cannot grow and morph.

Mohammad Razavi observes how a flow system can be found in the design of trees. In the article "Constructal Law of Vascular Trees for Facilitation of Flow" Razavi's explains that the evolutionary parameter is defined as the effectiveness of a tree structure to provide the most efficiency using the least amount of energy. In an experiment, conducted by Razavi, the characteristics of a tree's flow system were put into an equation to determine the functionality of a tree. It was found that the goal of a tree is to nourish its tissues through transporting nutrients, tree designs accomplishing this task may be at an evolutionary advantage because evolution favors efficient structures. The overall conclusions of this article were that tree structures adapt and morph to maximize flow through evolution (Razavi 14). Although Razavi's article uses a method full of mathematical proofs and equations to prove concepts of the constructal law, and Bejan uses a much more theoretical approach, they both reach the same conclusion: the structure of trees are designed to ease and increase flow. Razavi's piece proves Bejan's theories were correct; they are supported by what he found in his study about how trees facilitate flow because of the constructal law. As suggested by both of these specialists, evolution favors energy efficient structures, and to incorporate this, some natural structures use the Fibonacci Sequence within their designs.

The Fibonacci Sequence is present in many trees. This specific design facilitates flow within a tree's system (Bejan 141). The arrangement of branches around a trunk is commonly in the Fibonacci pattern, a design that helps trees and plants move water efficiently from the ground to the air. If trees follow the constructal law, use the Fibonacci sequence, and this design is a result of evolutionary refinement, it can be concluded that the Fibonacci sequence is one design used to increase efficiency and facilitate flow within trees and other living things.

Fibonacci Sequence in Plants

The Fibonacci sequence can be found in animal horns, mollusc shells, a variety spiral plants and other facets of nature (Livio 116-117). Many scientists have been fascinated by this mathematical guide that nature utilizes and they often try to discover why plants exhibit this pattern. In the article "Mathematical Lives of Plants," Julie Rehmeyer explains how researchers often suggest that the golden angle is used in plants because this design uses the least amount of stem space while also allowing maximum sunlight exposure (2). Although this can be proven through mathematical models, inferring that the golden ratio is found in plants because they are trying to receive the most sunlight is an illogical, oversimplified conclusion. Light hits plants in irregular ways, therefore, it is unlikely that this pattern would increase the amount of light hitting each leaf (Rehmeyer 2). With this conclusion also comes the assumption that the stems are always vertical and that leaves can't modify their position (Douady 272). Recent research suggests that even though 90% of spiraling plants have a relationship with the golden angle (137.5°), this phenomenon cannot be explained solely by the fact that plants try to absorb maximum light. The reason spiraling plants have their shape is attributed to a growth hormone called auxin that guides primordium (developing plant structures) towards the area with the least amount of growth hormone, creating an outward spiraling pattern.

The most efficient way to embed more structures in the least amount of space is achieved by utilizing the golden angle. This angle is where the least amount of growth hormone has built up on plants. Rehmeyer's article explains that plants may grow in certain patterns because of this growth hormone, but they do not discuss why. It could still be concluded that the reason plants produce the growth hormone that guides them, according to the Fibonacci sequence, is to provide the most exposure to resources. Maximizing the amount of primordium in a space by following the golden ratio is a result of evolutionary refinement that ensures efficiency.

Phyllotaxis

Phyllotaxis is the development of geometric patterns in plants, for example the arrangement of leaves on a stem. In an experiment conducted on the self organizing process of phyllotaxis a model was created to show how certain plants grow according to the Fibonacci sequence (Douady 255). It was proven that it is possible to obtain Fibonacci patterns through a physical experiment and a numerical simulation. The overall findings of the physical experiment in this research were that when drops of a magnetized liquid were put into a dish of oil, the drops repelled each other. Each drop would move at a phi angle (137.5º) in coherence to the drop before, which then turned into a Fibonacci spiral. Although this experiment was done with artificial materials, this pattern demonstrates how primordia move away from the central origin of a plant (Douady 272). This means that a plant's developing structures naturally repel each other at a 137.5º angle. Again, the reason this happens is due to a growth hormone that guides plant primordium to move outwards (Rehmeyer 2). Overall, it can be concluded that the

Fibonacci sequence is a self-arranging pattern plants often use to maximize space for new structures to grow.

Although the Fibonacci sequence appears in many plants, it is important to note that there are irregularities in these patterns and that not all plants use these specific patterns. Imperfections in the Fibonacci sequence can occur because of conditional or environmental factors such as distortion in surfaces grown in nature and in labs (Li 909). Plants grow where there is the most room for them, however, this does not always mean plants will grow in this exact pattern. For example some spirling plants grow according to a 99.5° angle, not 137.5º. This can be explained by the Lucas sequence: 1,3,4,7,11… where the following number is found by finding the sum of the previous two numbers (Rehmeyer 3). This is closely related to the Fibonacci sequence, but starts further along in the sequence with one instead of zero.

Countless plants grow with no relation to the golden ratio or the Fibonacci sequence. For example, some plant leaves grow opposite from each other. Although this can be explained by the biochemistry of growing plants, the Fibonacci sequence is not the answer to understanding why all plants grow the way they do or even why the entirety of living things evolve the way they do. It is unclear exactly how many plants in nature follow the Fibonacci sequence because it can be exhibited in many different forms. The constructal law and evolution of flow can better explore the complexities of all living things by explaining it with the goal of efficiency in mind (Bejan 3).

Patterns form in plants because of biochemical structures, the designs plants exhibit are unique and are not always connected to Fibonacci patterns. However, the patterns seen in plants that do exhibit the Fibonacci sequence have self-assembling characteristics which can be a useful tool in structural design (Douady 272).

Functionality of the Golden Ratio in our World

Self arranging patterns found through the Fibonacci sequence can be a helpful resource in the field of stress engineering. Chaorong Li observes how Fibonacci and triangular patterns seen in nature can be reproduced on a surface of microstructures in the article "Triangular and Fibonacci Number Patterns Driven by Stress on Core/Shell Microstructures" (909). These patterns appear through self-assembly with the purpose of minimizing total energy strain. Self-assembling patterns such as these are useful to material scientists. When these patterns are created they can be made simultaneously across large areas. This process allows for the creation of organized structures without high-precision patterning equipment (Li 909). This can be referred to as stress engineering. Fibonacci and triangular patterns are created by stress-driven, self-assembly, and surface tension. After conducting an experiment, Li discovered a fabrication technique that produces structures and patterns on a large area. Discovering the value of this pattern in stress engineering will allow more patterns to be created and used in the future.

Understanding the role of the Fibonacci sequence and the golden ratio can lead to a better understanding of nature and the human body. One specific human system that is heavily connected to the golden ratio is the cardiovascular system. Proportions derived from the golden ratio are found throughout this system, from the structure and size of the heart to the pattern in which it beats. A handful of studies suggest the golden ratio can be a tool for identifying a diseased heart early on. More specifically, if a heart does not follow the golden ratio proportions it is more likely to be diseased or have problems (Yalta 110). The ability to identify problems in a heart using the golden ratio can be combined with traditional methods to strengthen knowledge on the subject.

When organ systems are at their peak capacity and performance they tend to exhibit ratios proportional to the golden ratio (Yalta 108). This phenomena can be connected back the Fibonacci sequence in plants. Plants use the Fibonacci sequence to pack the most amount of seeds into a space and arrange leaves on a stem to allow for the most exposure to resources. It can be a sign that when natural systems are using this pattern successfully they reach peak functionality.

The reason humans are drawn to creating and studying patterns found in nature may be purely because of their functional value in human designs and understanding of organisms. The ability to do this consciously or unconsciously can suggest the beauty of the manifestation of the golden ratio in the world (Yalta 109). Adrian Bejan explains that “Through the much shorter annals of human history, our designs have evolved to create easier access for the flow of information to our brains, and to humanity at large” (299). Humans are drawn to progression and over time work towards progressing and increasing efficiency in designs. The Fibonacci sequence is useful in designs created to increase efficiency and in medicine. Humans are attracted to Fibonacci patterns because they can be helpful in advancements and understanding of ourselves and the world around us.

The Aesthetic Value of the Golden Ratio

The golden ratio is present throughout historical forms of art and architecture. Although the ratio can never be exact because phi(ϕ) is an irrational number, many notable masterpieces come suspiciously close. Concerning traditional styles of art, the golden ratio has manifested itself in many famous sculptures and paintings. Leonardo da Vinci used the golden ratio in much of his work, the most famous being the *Mona Lisa.* In this painting the golden rectangle can be used to highlight different facial features and proportions on her form (Atalay). Sculptures such as *Aphrodite of Melos* illustrate the golden ratio in the height of the figure divided by the height to the naval (Grigas 30). The golden ratio can even be found in ancient architecture forms such as the Great Pyramids in Giza and the Parthenon of ancient Greece (Grigas 29). There are many artistic works that embody the golden ratio, however, there are also many which do not.

It is still questioned whether or not artists intended to use the golden ratio in their work or if these proportions came up naturally because they are already manifested in the human body. The aesthetic value of the golden ratio has long been disputed. Even after centuries of discussion and experimentation no entirely conclusive results have been found. Aesthetics are concerned with beauty; perceptions of beauty vary between individuals. Studies on this topic are commonly not completely conclusive due to the difficulties of finding a universal meaning for beauty to use as the variable of an experiment.

Past Experiments

Christopher Green explores the more cognitive side to aesthetics in his article “All that Glitters: A Review of Psychological Research on the Aesthetics of the Golden Section.” Green analyzes a handful of different experiments done over the past 130 years to assess the validation of whether or not the golden ratio is aesthetically pleasing. Most of the experiments he studies consist of a scientist asking a series of test subjects to choose between a variety of proportioned shapes. Whether or not they choose the shape that is closest to the golden ratio to be the most visually appealing determines the results. Not all of the methods he studied conclude that the golden ratio is connected to aesthetics.

One specific set of experiments Green studied are those from Fechner in the 1860s, some of the first experiments done to determine the aesthetic value of the golden ratio. Fechner's method consisted of asking subjects to choose between differently proportioned rectangles to see which ones they found the most and least pleasing (Green 943). Although his results did not directly confirm the golden ratio was preferred, the overall trend pointed towards the golden ratio proportions being the ones to which people were most drawn. This is just one of many of Fechner's experiments that has been recreated time and time again; each experiment yielding varying results, some supporting and some contradicting Fechner's original conclusions and hypotheses.

After Green's wide analysis of experiments on this subject he concludes that "...there seems to be, in fact, real psychological effect associated with the golden section, but that they are relatively sensitive to careless methodological practices" (945). It is evident that there are a variety of techniques used to asses aesthetic value and a researcher's bias can greatly influence the outcome. If a researcher is looking for a connection hard enough they may force themselves to find it. The design of these experiments can also be discussed. Fibonacci patterns found in nature are found in thousands of shapes and forms, limiting their visual appeal to the proportions of a rectangle, as done in the experiment above, may limit the results. Aesthetics can be difficult to define, what is aesthetically pleasing to one viewer can easily be unappealing to another. Creating a test that relies on a variable this heavily opinionated could be flawed at the start. In general most past studies explain the difficulty of finding conclusions when looking at aesthetics because of the amount of room there is for interpretation and opinion.

The Golden Eye

One recent theory behind the appeal of the golden ratio is discussed in *Design in Nature* by Adrian Bejan. It is suggested that the shape of the human eye is a result of constructal design and its field of vision is proportionate to the golden ratio. This accounts for why humans are drawn to and sometimes create things similar to this shape. The design/shape of the human eye makes it so a form can be scanned horizontally faster then vertically. This is a result of the constructal law and evolution, and can be explained by the fact that humans live on a horizontal plane. When someone is given a set amount of time to scan something both horizontally and vertically, their field of vision results in a shape with proportions similar to the golden ratio. Bejan explains:

> The sublime beauty people find in objects that resemble the golden ratio is not due to some abstract quality that only the finest, most aesthetically attuned minds can appreciate. We consider them lovely and intriguing because they are in tune with how we see the world and are therefore useful. (229)

The goal of natural designs and many human creations is to flow efficiently. Images that are proportionate to our field of vision are easier to perceive and digest. This is a result of evolution and our urge to get the most out of something while using a minimal amount of energy. This does not mean shapes that are proportional to the golden ratio are visually “prettier.” Bejan argues that golden ratio shapes are easier for our brains to comprehend, which depending on how you define aesthetics can be an influential aspect of beauty. In comparison to Green’s conclusions on past experiments, one could ask if Bejan’s results would still be the same if he had run a series of physiological tests and if Green’s arguments would be further supported if he used Bejan’s mathematical approach.

The Golden Ratio and Visual Processing

Another theory on the aesthetic value of the golden ratio pertains to how fast humans can visually process images with Fibonacci patterns. In the article “The Golden Section as Optical Limitation” by Mark Elliott et al. an experiment was done to see if fast and efficient visual processing would be impaired and slowed when a test subject was presented with images that had golden ratio proportions. Participants were asked to perform in a series of tests that determine the speed and functionality of their visual processing. These experiments concluded that patterns with golden sectioning made visual processing less efficient which can be interpreted as creating an aesthetic response. Our visual system is very sensitive to spatial frequencies and the lack of these in golden section patterns slows response time (Elliott et al. 12). Sometimes an observer is aware of a slow visual processing caused by a certain pattern because of a particular environment such as a gallery. This environmental factor can causes a sense of aesthetic in itself. This hypothesis could be the reason the golden ratio has such a strong connection to aesthetics. However, this may require more studies to prove because ideas around aesthetics can be very interpretive (Elliott et al. 13).

It could be assumed that Elliott's argument comes in direct conflict with Bejan and his theory behind our field of vision. However, after analyzing both texts it can be concluded that each researcher is looking at different aspects of the golden ratio and each could have valid points. Bejan argues that because our field of vision is proportional to the golden ratio we can scan a shape that has these dimensions with ease since it is in tune with how we see our world. Elliot argues that patterns with sections proportionate to the golden ratio take longer to visually process than ones without and the extended period of time may be why we find the golden ratio visually appealing. Both of these texts have established that the golden ratio is a part of human nature and something that is connected to aesthetics. Bejan is looking at the overall shape of an object and its proportion to the golden ratio while Elliot looks at complex patterns that have the golden section within them. In a way these arguments are contradicting themselves because one is saying the golden ratio is easier for our brains to digest while the other is saying it takes longer. However, they are both using very different methods to get to their conclusions therefore neither should be completely disregarded. Although more research is needed to prove both of these concepts, combining the findings develops a new argument. Simple shapes with golden ratio proportions can be easier for our brains to process but as the degree of complexity increase visual processing becomes slower.

Part IV: Conclusions

Many natural patterns can be explained through the Fibonacci sequence, the constructal law, and the evolution of flow. Since evolution favors efficient structures, animate and inanimate forms in nature are constantly working towards becoming more efficient and using the least amount of energy to receive the most. The utilization of the golden ratio and Fibonacci sequence may be a sign that certain systems have reached their maximum efficiency. Humans are also a system working towards maximum efficiency and what we create may also be in coherence with the Fibonacci sequence and the golden ratio.

What can be learned from designs in nature should be used in human designs, especially when it concerns using self assembling pattern to lessen the energy strain on structures as done is stress engineering. The golden ratio can even be a tool to better understand the human body and systems such as the cardiovascular system. The connections between designs in nature, the designs in ourselves and the designs of our own creations suggest how truly connected we are to the natural world in which we live.

The reasons humans are visually drawn to patterns in nature and the golden ratio is still not entirely conclusive. Further studies are still needed to adequately understand the aesthetic value of this phenomenon. Two theories suggest that simple shapes connected to the golden ratio can be easier for our brains to process however, higher degrees of complexities may take longer for our brains to process.

If there is a connection between aesthetics and the golden ratio it should not be strictly integrated into the standards of art. Art is the process of expression and if patterns such as the golden ratio and Fibonacci sequence appear, they should be created naturally not because we have defined them to be the standard beauty. The Fibonacci sequence does not hold the restricted key to beauty, there are many beautiful patterns in our world that are still unaccounted for.

In the chaos of nature we can find patterns, some of these patterns follow the Fibonacci sequence and the golden ratio, yet there are many patterns which do not. The constructal law attempts to explain the rest of these patterns, but to develop an even deeper understanding of this topic it's necessary to also explore fractal dimensionality. Fractals describe the complexities and roughness on a surface and show that patterns in nature appear the same on a variety of scales. Fractal dimensionality is a mathematical way to explain chaos in nature and see relationships between the number of iterations within a pattern (Sanderson). We should not limit ourselves to solely understanding the value of the Fibonacci sequence, this pattern is only one example of a natural pattern that can be integrated into design.

It is important to note that the ideas in this discussion are not solidified, as this is only the beginning of an exploration. It is evident that the Fibonacci sequence exists in the natural world and some patterns found in nature can be explained through this phenomenon. Humans are drawn to these patterns and forms because they can cause an aesthetic reaction by affecting the brain's ability to process information and can serve as a tool in the world. When it comes to the functionality of the golden ratio, the Fibonacci sequence and other natural patterns, we should use nature as a guide to what we can do to become more efficient. Future implications of this could be useful in advancements in medical and design fields. Our knowledge of the Fibonacci sequence and the golden ratio should be used as a tool and as a base for further exploration, not as a limitation.

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